Some notes on electoral methods

1) The first aspect to dully consider is the fact that electoral methods do matter.

Imagine a situation where 3 candidates dispute a position (as M.P., President of the Republic or any other): Nader (let's suppose he is the first one) has 30 votes in a total of 100; Gore has 32 votes; finally, Bush has 38 votes.

If the method is the so-called "first-past-the-post", or mere "plurality", or "relative majority", Bush is immediately declared a winner.

There are very obvious problems, namely the fact that a vast majority of people (62, even considering only those who actually voted) opposes Bush.

If the method is a 2 rounds competition, as it often occurs with elections for PR, and in France also for the Parliament, Bush and Gore will accordingly dispute a second round.

If the second choices of the 30 people having voted for Nader are 22 for Gore and 8 for Bush, we thus have: Gore 54 votes (32 plus 22), Bush 46 votes (38 plus 8).

Therefore, in this case Gore is declared a winner: electoral methods do really matter a lot.

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2) However, things can get rather more complicated than this.

Suppose now that the criterion for choosing a winner is picking the one who is able to beat, in pairwise competition, or tournament, all the other ones.

Imagine also the following structure of preferences of voters:

Nader better than Gore, better than Bush, 22 votes;

Nader better than Bush, better than Gore, 8 votes (total of 30 for Nader as 1rst choice);

Gore better than Nader, better than Bush, 17 votes;

Gore better than Bush, better than Nader, 15 votes (total of 32 votes for Gore as 1rst choice);

Bush better than Nader, better than Gore, 22 votes;

Bush better than Gore, better than Nader, 16 votes (total of 38 votes for Bush as 1rst choice).

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In this case, Gore will beat Bush by 54 votes (32+22) against 46 (38 + 8);

on the other hand, Bush will beat Nader by 53 votes (38 + 15) against 47 (30 + 17);

and finally, Nader will beat Gore by 52 votes (30 + 22) against 48 (32 + 16).

Hence, it will be fair to say that social preferences, in this case, are not transitive:

Gore ≥ Bush ≥ Nader… but surprisingly Gore doesn’t beat Nader, instead Nader ≥ Gore!

This is called a “Condorcet paradox”, or in a slightly different wording a “Condorcet cycle”.

There is apparently no way here to choose a candidate that is all-winning in a pairwise tournament, and therefore indisputably the best.

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Important observation 1:

Notice that, if Nader represents a “Left” leaning, Gore is associated with “Centre” and Bush represents a “Right” leaning, the aforementioned structure of votes is rather incoherent and indeed very improbable. It is normal that the voters of Gore are divided by Nader and Bush as second options (17-15); but it is also rather unexpected that a considerable number of people from Nader’s ranks votes Bush as second choice (8), and above all that so many from Bush’s ranks opts for Nader as a second best (22)!

As a matter of fact, it is precisely this incoherence of choices, or alternatively the rankings not matching a global Left-versus-Right framing of perceptions, that determines the final paradoxical result.

In other words, it is the strong ‘personalization’ of confrontations, and particularly the deep, visceral aversion of Bush’s electorate for Gore, that ends up giving Nader a chance against him, and so inducing the global “cycle”, or “paradox”.

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Important observation 2:

Notice also that, more often than not, a ‘centrist’ candidate as Gore would have lots of chances. As a matter of fact, normally in 2 rounds competitions the ‘moderate’ ones, both on the Left and on the Right, have more chances in the second round: ‘radicals’ usually vote for ‘moderates’ as second choice, but frequently ‘moderates’ give up from ‘radicals’ (mostly by abstaining) if these are the ones who make it to the final round.

This puts an important evolutionary pressure into both Left and Right (parties and individual candidates alike), inducing them to promote everything ‘moderate’, and discard everything ‘radical’, thus tending to ‘normalize’ (in a statistical sense) the distribution of votes. This is sometimes called a “Duverger effect”, and it has affected during decades both Left and Right coalitions and formations, namely in France; but it also occurs very sharply in the USA via the so-called “primaries”, that basically operate as a first round of elections, but proceeding only ‘internally’ to each of the 2 big parties: also usually enhancing ‘moderates’ and chastising ‘radicals’.

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3) However, even assuming that ‘centrist’ candidates tend to be benefited by a 2 rounds system of counting, it may also occur that it becomes impossible for them to get elected, simply by not managing to do it to the second round.

Imagine a candidate A on the Left, getting 40 votes, a candidate B on the center with 15 and a candidate C on the Right wing with 45 in the first round. By a first-past-the-post method, C gets immediately elected.

If there is a second round, it may occur the second choices of B voters are: 12 for A and 3 for C. Ergo, in this case we have a final outcome of: A gets 52 votes, and C gets 48 votes; A wins.

In spite of all this, notice that in a putative second round against A, the candidate B would likely get 60 votes (45 for C, plus 15 ‘directly’ for B) against 40 for A; and in a match B versus C things would normally be: candidate B obtains 55 votes (15 ‘directly’, plus 40 having previously voted for A) and C obtains 45 votes. Hence B wins both matches.

Therefore, B is clearly a “Condorcet-winner”.

However, the fact is that he/she never gets to be elected: neither by the ‘first-past-the-post’ method, nor by a 2 rounds method.

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Besides being a Condorcet-winner, B would also triumph in a competition taking into considerations a ranking of choices. If we assume 2 votes for the first choice, and 1 for the second, we have:

A – 40x2 + 12 = 92 votes;

B – 15x2 + 40 + 45 = 115 votes;

C – 45x2 + 3 = 93 votes.

This is called a “Borda counting”, and in this case B would also win.

He is, therefore, simultaneously a “Condorcet winner” and a “Borda winner”.

Yet still, he doesn’t get elected: ‘first-past-the-post’ or 2 rounds method.

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One more important observation:

it was noted, much after Marie-Jean Condorcet and Jean-Charles Borda (both late 18th century authors), by Kenneth Arrow (a 20th century economist) that all methods of counting based on equal (non-weighted) suffrage are prone to produce either situations of intransitivity (i.e. Condorcet paradox), or situations where “irrelevant alternatives” (the presence or absence of 4th, 5th candidates…) and “strategic vote” induce changes in the final outcome of elections.

In other terms, there is not a completely ‘perfect’ method: those immune to strategic vote and irrelevant alternatives are prone to intransitivity, and those immune to intransitivity are prone to suffer the other ‘diseases’.

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4) In a large measure with the purpose of avoiding the problems associated with all forms of counting producing just one winner, various other methods were invented, taking into consideration the will of minorities and the *dégradé* of individual choices.

One of them was the constitution of plurinominal circles, using a mixture of ‘quota’ and ‘bigger remains’ methods.

Imagine a circle producing 4 MPs. If que quota is $1/(n+1)$, any candidate obtaining 20 per cent, or 1/5 of the votes, gets immediately elected.

After that, if no more candidates attain 20 per cent, we pick up those who are closer to that, or in other terms use the ‘bigger remains’.

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Imagine now a situation where:

A gets 50 per cent; B gets 16 per cent; C gets 15 per cent; D gets 10 per cent; E gets 4 per cent; F gets 3 per cent; and finally G gets 2 per cent.

Obviously, A is immediately elected by the quota system, but 30 per cent of votes are wasted in this case. Quite a grotesque excess!

After that, B, C and D will also be chosen, using the so-called method of ‘bigger remains’.

Therefore: 50 per cent of the vote produce 1 M.P. (A), and 10 per cent produce another one (D).

Evidently an unfair outcome!

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In order to avoid these discrepancies, a number of other methods were suggested.

One of them is the so-called "single transferable vote" method, namely the one proposed by Thomas Hare. Suppose the voters of A have indicated second preferences, 2/3 of them opting for E, and 1/3 for D. Therefore, and given the fact that there are 30 per cent of 'excessive' votes in A, we can give an extra 20 to E and an extra 10 to D. In this case, E has now 24 per cent and D has 20 per cent: both get automatically elected.

Subsequently, we take the now 'extra' 4 per cent of E, checking whether there are third preferences indicated, or not. In case there are not, we eliminate the smaller candidates, F and G, and use their votes, checking the possibility that some indicate B or C as second choice. Supposing that all of them mention C as second best, than C gets 15+3+2, that is, 20 per cent; he thus is also elected.

Hence, the list of the elected candidates is: A, E, D and C.

On the other hand, B remains with 16 per cent only… and therefore he is out.

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5) One more radical variant of this procedure are the truly proportional methods, whereby the transfer of ‘extra’ votes from the first candidate in one list into the second, the third, etc. is assumed, or taken for granted.

To repeat, candidates now run in lists, and if the double of what is needed to elect one M.P. is obtained, then the second of the list is automatically assumed to be also elected, and so on.

Probably to most ‘classical’ of these methods is the one proposed by Jacques d’Hondt. According to Hondt’s method, we divide the total number of votes obtained by each list by 1, and afterwards by 2, 3, 4, etc. The elected MPs correspond to the higher values in the matrix, starting from top left, and reading downwards and to the right.

Let us consider the following example. If our circle produces 10 MPs and the list A has 195 votes, followed by list B with 120, list C with 100 and so on, then A will get 3 MPs, B and C will obtain 2 each, whereas D, E and F will acquire 1 each, and finally G gets none.

With big circles, Hondt method produces an almost perfect proportionality, but with small circles it tends to hurt small parties. As a matter of fact, in our case the list F, with a bit less than 9 per cent of the votes, is lucky to obtain 1 in 10 MPs; oppositely, list G, with roughly 6.5 per cent of the suffrages, is left out of the Parliament.

Hondt method

	A	B	C	D	E	F	G	H
1	195,0	120,0	100,0	80,0	70,0	60,0	45,0	30,0
2	97,5	60,0	50,0	40,0	35,0	30,0	22,5	15,0
3	65,0	40,0	33,3	26,7	23,3	20,0	15,0	10,0
4	48,8	30,0	25,0	20,0	17,5	15,0	11,3	7,5
5	39,0	24,0	20,0	16,0	14,0	12,0	9,0	6,0
6	32,5	20,0	16,7	13,3	11,7	10,0	7,5	5,0
7	27,9	17,1	14,3	11,4	10,0	8,6	6,4	4,3
8	24,4	15,0	12,5	10,0	8,8	7,5	5,6	3,8
9	21,7	13,3	11,1	8,9	7,8	6,7	5,0	3,3
10	19,5	12,0	10,0	8,0	7,0	6,0	4,5	3,0

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Important variations from the model represented by Hondt method are the Imperiali method and the method of Sainte-Laguë.

With Imperiali method, the line correspondent to 1 is not considered, which implies that the first MP is harder to conquer, and thus the smaller parties are hindered. The purpose is therefore, so to speak, to have proportionality *ma non troppo*…

With Sainte-Laguë method, oppositely, the purpose is to supply a 'head-start' to small parties. Therefore, in the count we use 0.5 - 1.5 - 2.5 – 3.5… which means that the first MP is easier to get (it costs only a half of the following ones), whereas all the subsequent cost a normal 'price' referring to number of votes. Obviously, instead of the series made of 0.5 - 1.5 - 2.5 – 3.5… we can also use 1-3-5-7…, or in other terms only the odd numbers.

Comparing the outputs, we can easily verify that, in the Imperiali situation, A gets 1 extra MP and F loses its one.

Oppositely, in Sainte-Laguë's case C ends up losing its second MP, whereas G is *par contre* allowed to step in.

Imperiali method

	A	B	C	D	E	F	G	H
1	195,0	120,0	100,0	80,0	70,0	60,0	45,0	30,0
2	97,5	60,0	50,0	40,0	35,0	30,0	22,5	15,0
3	65,0	40,0	33,3	26,7	23,3	20,0	15,0	10,0
4	48,8	30,0	25,0	20,0	17,5	15,0	11,3	7,5
5	39,0	24,0	20,0	16,0	14,0	12,0	9,0	6,0
6	32,5	20,0	16,7	13,3	11,7	10,0	7,5	5,0
7	27,9	17,1	14,3	11,4	10,0	8,6	6,4	4,3
8	24,4	15,0	12,5	10,0	8,8	7,5	5,6	3,8
9	21,7	13,3	11,1	8,9	7,8	6,7	5,0	3,3
10	19,5	12,0	10,0	8,0	7,0	6,0	4,5	3,0

Sainte-Laguë method

	A	B	C	D	E	F	G	H
1	195,0	120,0	100,0	80,0	70,0	60,0	45,0	30,0
2	97,5	60,0	50,0	40,0	35,0	30,0	22,5	15,0
3	65,0	40,0	33,3	26,7	23,3	20,0	15,0	10,0
4	48,8	30,0	25,0	20,0	17,5	15,0	11,3	7,5
5	39,0	24,0	20,0	16,0	14,0	12,0	9,0	6,0
6	32,5	20,0	16,7	13,3	11,7	10,0	7,5	5,0
7	27,9	17,1	14,3	11,4	10,0	8,6	6,4	4,3
8	24,4	15,0	12,5	10,0	8,8	7,5	5,6	3,8
9	21,7	13,3	11,1	8,9	7,8	6,7	5,0	3,3
10	19,5	12,0	10,0	8,0	7,0	6,0	4,5	3,0